

Some basic Formulas

Nitrogen (N₂), Oxygen (O₂), Carbondioxid (CO₂), Argon (Ar), Helium (He), ...

Index t (Index _t) for tissue = actually this is not a real body tissue but rather a compartment!

That is, a mathematical entity which symbolises a group of various body tissues which share common properties, namely the half-time (that is basically the inverse of the perfusion) and solubilities.

Index amb (index _{amb}) for ambient

Relationship between partial pressures of various gases and the total ambient pressure $P_{amb} = P$ (Dalton's Law):

$$P = \sum p_i = p_1 + p_2 + p_3 + \dots \quad (51)$$

At sealevel this is approx.:

$$P = \sum p_i = p_{O_2} + p_{N_2} + p_{Rest} = 1 \text{ Bar (ca.)}$$

In order to calculate the partial pressures p_i we need the portion in the volume f (= fraction) of each of these gases:

$$p_i = P_{amb} * f_i$$

Example for regular air at sealevel (ca.):

$$p_{O_2} = 1 \text{ Bar} * 0,2$$

$$p_{N_2} = 1 \text{ Bar} * 0,8$$

Units of Length

$$1 \text{ m} = 1,094 \text{ yard} = 3,28 \text{ feet} = 39,37 \text{ inches}$$

$$1 \text{ foot} = 0,3048 \text{ m} = 12 \text{ inch}$$

$$1 \text{ inch} = 2,54 \text{ cm}$$

$$1 \text{ fathom} = 6 \text{ ft} = 2 \text{ yard}$$

$$3 \text{ feet} = 1 \text{ yard} = 0,91439 \text{ m}$$

Units of Pressure

$$1 \text{ Bar} = 10^5 \text{ Pascal (Pa)} (= \text{N} / \text{m}^2) = 14,5038 \text{ psi}$$

$$1 \text{ Atmosphere (atm)} = 760 \text{ mm Hg (Torr)} = 10,080 \text{ Meter Sea Water (msw)} = 10,33227 \text{ m WS} =$$

$$33,071 \text{ feet sea water (fsw)} = 33,8995 \text{ feet fresh water} = 14,6960 \text{ psi}$$

$$1 \text{ fsw} = 0,0306391 \text{ Bar}$$

$$1 \text{ foot of pure water} = 0,0298898 \text{ Bar}$$

$$1 \text{ Atmosphere (atm)} = 1,03323 \text{ kg/cm}^2 \text{ (bzw.: } 1.033,23 \text{ cm H}_2\text{O)} = 1,01325 \text{ Bar} (= 1013,25 \text{ mbar)} =$$

$$760 \text{ Torr (mmHg)} = 101,325 \text{ Kilopascal (kPa)}$$

$$1 \text{ fsw} = 22,9809 \text{ mm Hg}$$

$$1 \text{ psi} = 0,0689474 \text{ Bar}$$

Relationship between a diving depth d , measured in a pressure unit for eg. [bar] or [fsw] and the absolute ambient pressure P_{amb} and the pressure at sea level (p_0):

$$P_{amb} = p_0 + d$$

in general:

$$P_{amb} = p_0 + d * \text{const.}$$

This constant is for conversion from units of length (diving depth) to units of pressure.

If you have the diving depth d in meters [m], there is approx. this relation with the pressure [Bar]:

const. = 0,0980665 [Bar/m], for fresh water and

const. = 0,100522 [Bar/m], for sea water, and thus for eg.:

$$P_{amb} = p_0 + d / 10,197$$

resp.:

$$P_{amb} = p_0 + d / 9,948$$

References for the conversion factors, for e.g.:

[75], p. 575; [63], p. 457; and especially [110], p. 893! As well: [167], p. 354 - 358

The various citations of books here in this file, like [yxz] refer to:

[the corresponding entry in our literature list here on divetable.info](#)

General Remarks

The on- and off-gasing with inert gases of a system which is not in equilibrium with these gases is best described with a little differential equation:

$$dP_t(t)/dt = k[P_{alv}(t) - P_t(t)] \quad (1a)$$

Descriptively this formula means: the pressure change over time $dP_t(t)/dt$ within a compartment is dependant of the actual pressure gradient ΔP . So the basic mathematical idea is simply like that:

$$\partial P / \partial t \sim \Delta P$$

From this proportionality we make a real equation with the help of a simple constant; and the ∂ s become real differentials:

$$dP(t)/dt = k * \Delta P$$

The central idea of all so-called **perfusion models** is an instantaneous pressure equalization from the alveoli in the lung with the arterias!

I.e.: $P_{alv}(t) = P_{arterial}(t)$, for all t

Variable	Definition
$P_t(t)$	Partial Pressure of an inert gas within a compartment at time t, in [Bar]
$P_{alv}(t)$	Partial Pressure of an inert gas within the alveoli [Bar]
k	a constant, for each compartment [min^{-1}]
t	time (as well dive time) [min]

Step 1: We are separating the variables in (1a) and find the inhomogeneous differential equation (1b):

$$dP_t(t)/dt + kP_t(t) = kP_{alv}(t) \quad (1b)$$

Step 2: The homogeneous equation (2):

$$dP_{th}(t)/dt + kP_{th}(t) = 0 \quad (2)$$

For the solution we take the following approach: $P_{th}(t) = C_0 e^{-\lambda t}$

The subscript 'h' in P_{th} means the homogeneous equation. We put this into (2) and get:

$$-\lambda C_0 e^{-\lambda t} + k C_0 e^{-\lambda t} = 0 \Rightarrow (k - \lambda) C_0 e^{-\lambda t} = 0,$$

only if $k = \lambda$ or $C_0 = 0$ (uninteresting: it means no change in pressure, i.e.: no dive),

and because $e^{-\lambda t} \neq 0$ for all t, we take $k = \lambda$ and thus getting the homogenous solution for (2):

$$P_{th}(t) = C_0 e^{-kt}. \quad (3)$$

Step 3: Find a special solution of the inhomogeneous equation (1b) through the boundary conditions. We are taking two special situations concerning $P_{alv}(t)$.

1) $P_{alv}(t) = \text{const.}$, i.e. the depth as well is constant!

2) $P_{alv}(t)$ varies linearly in time, i.e. the ascent- and descent-ramps are constant, that is the ascent- and descent velocities remain constant.

Situation 1:

constant depth = constant ambient pressure

In this case the alveolar inertgas pressure will take a constant value: $P_{alv}(t) = P_{alv0}$. So the equation becomes (1b):

$$dP_t(t)/dt + kP_t(t) = kP_{alv0} \quad (4)$$

We are taking the 'solution':

$$P_t(t) = C_0 e^{-kt} + C_1 \quad (5)$$

we put (5) into (4) and the exponential terms are cancelling themselves:

$$-k C_0 e^{-kt} + k C_0 e^{-kt} + k C_1 = k P_{alv0}$$

thus we receive: $C_1 = P_{alv0}$.

We are searching another boundary condition in order to determine C_0 .

We assume a certain but constant partial pressure at start in the compartment: $P_t(0) = P_{t0}$ for time $t=0$. This means for eg. a certain reached saturation pressure or as well a starting pressure for mountain lake diving. We put his into equation (5):

$$P_{t0} = C_0 e^{-0} + P_{alv0}$$

and thus follows: $C_0 = [P_{t0} - P_{alv0}]$.

We will have then the **Haldane** equation for the partial pressure of an inert gas in a specific compartment k:

$$P_t(t) = P_{alv0} + [P_{t0} - P_{alv0}] e^{-kt} \quad (6a)$$

Variable	Definition
$P_t(t)$	partial pressure of an inert gas within a compartment with the constant k [Bar]
P_{t0}	initial partial pressure of an inert gas within a compartment at time $t=0$ [Bar]
P_{alv0}	the constant partial pressure of an inert gas within the alveoli [Bar], for $t = 0$ and therefore for all times t due to the boundary condition
k	a constant, dependant of compartment [min^{-1}]
t	time [min]

(6a) is the famous **Haldane equation**, we re-arrange and arrive at (6b);

we simply add at the left and right side P_{t0} and group according $P_{alv0} - P_{t0}$:

$$P_t(t) = P_{t0} + [P_{alv0} - P_{t0}] [1 - e^{-kt}] \quad (6b)$$

This is the equation which is regularly cited, e.g.:

Bühlmann, [4] p. 14, 1983; [5] p. 64, 1993; [65] p. 96, 2002, with slightly changed indices.

For e.g. P_I is the inspiratory inert gas pressure, this means the respiratory coefficient R_q (pls. cf. equation (13)) is implicitly = 1,

thus we have variances across different tables/models/researchers: have a look at the next chapter.

In [4] on p. 60 we could see this very clearly.

Situation 2:

the ambient pressure changes linearly with time

During ascents and descents with constant velocity the inspiratory partial pressure of the inert gases changes linearly with time (with an open circuit SCUBA! With rebreathers (CCR = closed circuit rebreather) we have, according to make, other dependencies!). In equation (1) this implies:

$$P_{alv}(t) = P_{alv0} + R \cdot t.$$

P_{alv0} is the partial pressure of the inert gases from start, i.e.: at time $t=0$, and R is the rate of change (in Bar/minute) of the partial pressure of the inert gases in the alveoli. R is positiv for descent (increase of pressure) and negativ for ascent (decrease of pressure). We put it into equation (1b):

$$dP_t(t)/dt + k P_t(t) = k P_{alv0} + k R t \quad (7)$$

We try the solution:

$$P_t(t) = C_0 e^{-kt} + C_1 t + C_2 \quad (8)$$

We plug solution (8) into equation (7) and receive:

$$-k C_0 e^{-kt} + C_1 + k C_0 e^{-kt} + k C_1 t + k C_2 = k P_{alv0} + k R t$$

and thus:

$$[k C_1 - k R] t + [C_1 + k C_2 - k P_{alv0}] = 0 \quad (9)$$

In order to find a solution for C_1 and C_2 which should be valid for all times t , we put the terms in the squared brackets in (9) equal to 0. Thus we have:

$C_1 = R$ and $C_2 = P_{alv0} - R/k$. Thus we have the following:

$$P_t(t) = C_0 e^{-kt} + R t + P_{alv0} - R/k \quad (10)$$

Once more we exploit the boundary condition $P_t(0) = P_{t0}$ for $t=0$ in order to calculate C_0 . We put this into (10) and get:

$$P_{t0} = C_0 e^{-0} + P_{alv0} - R/k$$

and thus: $C_0 = P_{t0} - P_{alv0} + R/k$. As a definite solution we find:

$$P_t(t) = P_{alv0} + R \cdot t - R/k + [P_{t0} - P_{alv0} + R/k] e^{-kt}$$

resp.:

$$P_t(t) = P_{alv0} + R \cdot [t - 1/k] - [P_{alv0} - P_{t0} - R/k] e^{-kt} \quad (11)$$

Variable	Definition
$P_t(t)$	partial pressures of the inert gases within compartment [Bar]
P_{t0}	initial partial pressures of the inert gases in compartment at time $t=0$ [Bar]
P_{alv0}	initial (alveolar) partial pressure at time $t=0$ [Bar]
k	a constant, dependant of compartment
R	rate of change of the partial pressure of an inert gas in the alveoli (Bar/min) $R = f \cdot R_{amb}$, f is the fraction, i.e.: volume part of inert gase and R_{amb} is the rate of change of the ambient pressure
t	time [min]

the so-called **Schreiner equation**.

By putting the rate of change $R = 0$ (constant diving depth) the Schreiner equation (11) becomes again the Haldane equation (6a).

Half Times (HT)

The variable τ (a greek Tau) is designated as a 'Half Time' (HT) and is specific for each compartment:

$$-k * \tau = \ln(1/2) = -\ln(2)$$

The relation between k and the HT is:

$$\begin{aligned} \tau &= \ln(2) / k \\ \text{resp.: } k &= \ln(2) / \tau \end{aligned} \quad (12)$$

The stringent physiological relation between the HT and the compartments is through the solubility and the perfusion (blood supply):

$$\tau = 0,693 * \alpha_{ti} / (\alpha_{bl} * dQ/dt) \quad (12a)$$

with the following definitions:

$$\begin{aligned} \alpha_{ti} &= \text{solubility of an inert gas in tissue, } \text{ml(S)}_{\text{gas}} * \text{ml}_{\text{ti}}^{-1} * (100 \text{ kPa})^{-1} \\ \alpha_{bl} &= \text{solubility of an inert gas in blood, } \text{ml(S)}_{\text{gas}} * \text{ml}_{\text{blood}}^{-1} * (100 \text{ kPa})^{-1} \\ dQ/dt &= \text{compartment perfusion, } \text{ml}_{\text{blood}} * \text{ml}_{\text{ti}}^{-1} * \text{min}^{-1} \end{aligned}$$

pls. cf. further down for a rationale of this relationship

The partial pressures in the lung alveoli

we want to determine the alveolar partial pressure P_{alv} more exactly. The gas composition and thus the partial pressures of the component gases are dependant of the ambient pressure P_{amb} , water vapour, carbon dioxid, and the other components of the breathing mix, i.e. the chemical composition.

The ambient pressure P_{amb} is the sum of the atmospherical pressure (ca. 1 Bar at sea level) + static pressure of the water column (from the diving depth), thus it is increased ca. 1 Bar per every 10 m increase in depth (not considering the differences between fresh and sea water ...). The ambient pressure and the absolute pressure within the lung have to be (roughly) equal, only through in- and exhalation there is a small pressure difference of approx. and up to max. 30 cm of water column.

The partial pressure of an inert gas in the lung P_{alv} can be roughly estimated through:

- the partial pressure (through the fraction f) of the inert gas in the air or in the mix
- the water vapor partial pressure P_{H_2O} . The dried air from the compressor has to be humidified on the way to the lungs. The nose, voice box, tracheas and various mucus membranes put their water vapor into the dry air: thus the breathing mix gets diluted. At 37 degrees of Celsius the P_{H_2O} is in saturated environment ca. 0.0627 Bar (47 mm Hg)
- Oxygen O_2 decreases through metabolism and ventilation with used air
- Carbon dioxid CO_2 is being added through metabolism / ventilation
- and because CO_2 in normal clean breathing air can be neglected, we put CO_2 partial pressure in the lung alveoli equal the arterial CO_2 partial pressure with: 0.0534 Bar (40 mm Hg).

The relation of the oxygen consumption to the carbon dioxid production is designated with the **respiratory quotient** R_q .

R_q is the proportion in volumes of carbondioxid production to oxygen consumption in ml, average values are:

200 ml carbondioxid production / oxygen consumption 250 ml per minute, i.e. ca.:

$$R_q = 200 / 250 = 0,8$$

R_q is dependant on nutrition and on the workload:

We have $R_q = 0.7$ for metabolism mainly on fat, ca. 0.8 for proteins, ca. 0.9 – 1 with carbohydrates;

With average workload and nutrition we have 0,8 -> 0,85, with CO_2 retainers < 0,7

and > 1,0 with heavy workloads or acidosis/enrichment with lactic acid.

The equation of the **alveolar ventilation** describes this:

$$P_{alv} = [P_{amb} - P_{H_2O} - P_{CO_2} + \Delta P_{O_2}] * f$$

or:

$$P_{alv} = [P_{amb} - P_{H_2O} + (1 - R_q) / R_q * P_{CO_2}] * f \quad (13)$$

Variable	Definition
P_{alv}	partial pressure of the inert gas in the alveoli [Bar]
P_{amb}	ambient pressure, i.e. absolute pressure of breathing mix [Bar]
P_{H_2O}	water vapor partial pressure, at 37 degrees of Celsius ca. 0.0627 Bar (47 mm Hg)
P_{CO_2}	carbon dioxid partial pressure, ca. 0.0534 Bar (40 mm Hg)
ΔP_{O_2}	Delta = change of oxygen partial pressure through metabolism / ventilation in the lung
R_q	the respiratory quotient
f	fraction of inert gases in the breathing mx; e.g.: N_2 in dry air = ca. 0.78. Normally for diving we put $f = 0,79xx$, i.e.: we take into account the various trace gases

Across the various models/researchers we have as well various settings for the R_q , even if not explicitly stated:

Schreiner: $R_q = 0.8$

Workman: $R_q = 0.9$

Buehlmann / Hahn: $R_q = 1.0$

critical supersaturation, symptomless tolerated inert gas overpressure, M-Values

Haldane

The tolerated overpressure is postulated as 2 : 1 for all 5 compartments.

These 5 compartments had the HT: 5, 10, 20, 40, 75 min. The 2:1 ratio is valid only for the absolute pressure!

Concerning the inert gas overpressures in relation to the ambient pressure we have the following:

$$2 * 0,78 = 1,56, \text{ i.e.: } 1,56 : 1 \text{ (elucidation from Workman)}$$

and is a constant for all compartments.

Workman

The linear Workman equation, is a simple straight line in a pressure coordinate system. It looks like that for each compartment:

$$M = M_0 + \Delta M * d \quad (14)$$

Variable	Definition
M	M-Value, maximal partial pressure of an inert gas in a compartment [fsw]
M_0	M-0 Value, for sea level or diving depth = 0 ft for each compartment [fsw]
ΔM	Delta M, increase of M per each foot of diving depth, defined for each compartment [fsw/ft]
d	diving depth [ft]

We have compiled [all these coefficients in a separate file](#) so that you can copy-&-paste it directly to your own DIY deco procedures.

Bottom Line: Workman had the M-Values decreasing with increasing HT, thus the fast compartments (short HT) can tolerate a higher inert gas overpressure than the slower compartments (longer HT).

With equation (14) we could determine the minimal depth d_{\min} where the diver has to stay during his/her deco stops. The calculation we have to do for each of the compartments and is dependant of the topical inert gas overpressures:

$$d_{\min} = (P_t - M_0) / \Delta M \quad (15)$$

So each compartment has got its own saturation and thus its own minimal depth. We simply have to take the biggest value (greatest depth) out of the family of these values.

Between the M-values and the HT we have approximately the following empirical relation according to Workman:

$$M = 152.7 \tau^{-1/4} + 3.25 d \tau^{-1/4} = M_0 + \Delta M d \quad (16)$$

According to Hempleman we have an approximate empirical relation between the no-decompression limits t_{NDL} [min] and the diving depth d [fsw]:

$$d * t_{\text{NDL}}^{1/2} = 475 \text{ fsw min}^{1/2} \quad (16.1)$$

The PADI/DSAT RDP table has been generated with such a formula, but with a couple of more conservative modifications:

$$d - A = C * t_{\text{NDL}}^{-x} \quad (16.2)$$

- A = "shallow" asymptote, 20.13 fsw
- C = 803
- x = 0.7476
- "deep" asymptote = 262 fsw (theoretical diving depth with NDL = 0!)

Buehlmann / Hahn

Buehlmann offers, very like Workman, a linear relationship between the tolerated tissue overpressure and ambient pressure ([65], p. 117):

$$P_{t,\text{tol.ig}} = P_{\text{amb}} / b + a \quad (17)$$

Variable	Definition
$P_{t,\text{tol.ig}}$	tolerated inert gas pressure, for each compartment, (analogous M) [Bar], sum of all partial pressures of the present inert gases
a	boundary value at a theoretical ambient pressure of 0 Bar, i.e. the axis intercept [Bar]
P_{amb}	ambient pressure, absolute pressure of the breathing mix [Bar]
b	1/b pressure gradient: value of increase per pressure unit depth (dimensionless), i.e. the slope of the straight line

Basically there is no difference between Workman and Buehlmann / Hahn except that the M-values are referenced to the ambient pressure at sea level whereas Buehlmann / Hahn are extrapolating against 0 Bar ambient pressure and thus reaching the region between 1 and 0 Bar automatically, i.e. mountain lake diving. You could convert both systems into eachother:

$$\begin{aligned} P_{t, \text{tol.ig}} &= M \\ \Delta M &= \text{const.} * 1/b \\ M_0 &= a + p_0/b \end{aligned} \quad (18)$$

According to Buehlmann and co-workers (Keller et al ...) there is the following empirical relationship between the a- and b- coefficients to the HT. This one is valid only for nitrogen. ([65], p. 129)!

For helium it looks a little bit different ... (pls. cf. [65], S. 131).

$$\begin{aligned} a &= 2 \text{ Bar} * \tau^{-1/3} \\ b &= 1.005 - \tau^{-1/2} \end{aligned} \quad (19)$$

This is the so-called set "A"; further on there is a set called "B" (more conservative, for printed tables) and as well a "C" set for on-line calculations with a diving computer ([65], p. 158).

The ratio of the HT of two immediately sequenced compartments is roughly 1.2 to 1.4.

(Workman for eg. puts this to another value: $5 * 2^n$, n being the compartment no. ...)

This is not a strict physiological law but respects the desire to have a narrowly cramped network of the HT. Only the very fast and the extremely slow HT are representing compartments which have been verified through a lot of experiments on humans in the pressure chamber in Zuerich in the 60's. Nevertheless there is a coarse one-to-one mapping of HT with "real" body tissues from p. 115 in [65]:

- Nr. 1 - 4: brain and spinal cord
- Nr. 5 - 11: skin
- Nr. 9 - 12: muscles
- Nr. 13 - 16: joints, cartilage, bones
- Nr. 7 - 16: internal ear

The cited root to the third from the derivation of the coefficient a pinpoints the tolerated surplus volume of the inert gas.

A little bit more handy is the following, from (17):

$$P_{\text{amb, tol}} = (P_{t, \text{ig}} - a) * b$$

other perfusion models

DCAP (Decompression Computation and Analysis Program) is using the M11F6 M-Values, developed by Bill Hamilton et al. for the swedish navy. The 11 HT for nitrogen are from 5 - 670 min., and therefor these for helium: 5 - 240 min.

The **PADI RDP™** coefficient set uses the 14 M-Values developed by Raymond E. Rogers and Michael R. Powell, Diving Science and Technology Corp (DSAT). With this model, YOU KNOW IT, you are restricted to only NDL dives! There are no validated data outside the recreational NDL envelope! **I.e.: these coefficients are NOT for decompression diving and as well not for mountain lakes!**

diffusion model

The **RNPL SLAB** Model:

RNPL is the "Royal Naval Physiological Laboratory" of Her Majesty, the Queen.

A SLAB means a piece of tissue. On one side the slab is in contact with blood, so there is perfusion. By diffusion the inert gas molecules are moved through this model-tissue. These SLABS have been put into the BLACKPOOL tables for caisson-workers and subsequently into the BSAC 1988 tables

$$\begin{aligned} P_1 &= P_{\text{ambient}} - 8 / \pi^2 * (P_{\text{ambient}} - P_0) * (e^{-k*t} + 1/9 * e^{-9*k*t} + 1/25 * e^{-25*k*t}) \\ r &= 27.5714 / P_1 + 12.407 \\ r &= \text{Supersaturation Ratio, } 1.6 < r < 1.9 \\ P_1 &\text{ in Bar} \\ k &= 0.007928 \end{aligned}$$

This is a special solution of the 2nd Fick equation (yes, yes: that was really the guys name ...) For the diffusion coefficient D we have:

$k = D * \pi^2 / 4 * b^2$, b is the thickness of the slab.

Das **DCIEM Kidd-Stubbs** model:

DCIEM = Defence and Civil Institute of Environmental Medicine, Canada:

In this model we have **4 compartments in series**, i = 1 - 5.

This is the model of Kidd and Stubbs from 1962 resp. the "KS-1971".

The DCIEM tables from 1983 (R.Y. Nishi and G.R. Lauckner) a relying on it.

$$\begin{aligned} dp_i / dt &= A * ((B + P_{i-1} + P_i) * (P_{i-1} - P_i) - (B + P_i + P_{i+1})(P_i - P_{i+1})) \\ \text{with: } P_0 &= P_{\text{ambient}} \\ \text{and: } P_i &= P \text{ in the compartment } i \\ \text{and: } P_5 &= 0 \\ A &= 0.0002596 \\ B &= 83.67 \\ A \text{ and } B &\text{ are the so-called "flow constants" for air, in msw} \\ \text{in imperial units we have for air:} \\ A &= 79.12 * 10^{-6} \\ B &= 274.5 \\ \text{with other mixes, for e.g.: Heliox10 (90 \% Helium) yields:} \\ A &= 66.48 * 10^{-6} \\ B &= 611.7 \end{aligned}$$

The HT themselves are defined via the A- and B-coefficients and pressure- resp. time-dependant.

The asymmetry between inert gas absorption (faster) and -release (slower) can be seen here:

$$T_{1/2} = (\ln(2 - \Delta P / (B + P_i + P_f))) / (A(B + 2P_f))$$

with: P_i initial pressure

mit: P_f finale pressure

$$\Delta P = P_f - P_i$$

with: SAD = Safe Ascent Depth

$$SAD = P_{\max, \text{Komp}, i} / 1.8 - 10.06$$

for the DCIEM table we have the following:

$$SAD = P_{\max, \text{Komp}, i} / R - \text{OFF} - P_{\text{scalelevel}}$$

$$i = 1, R = 1.300 \text{ and } \text{OFF} = 4.8$$

$$i = 2, R = 1.385 \text{ and } \text{OFF} = 2.5$$

$$P_{\text{scalelevel}} = 10.06 \text{ msw}$$

and for $i = 3; 4; 1/R = 0.0$ and, as well $\text{OFF} = 0.0$

for the half-times with the above values and:

$$T_{1/2} = \ln(2) / A * B$$

AIR:

$$T_{1/2} = 31.91 \text{ min}$$

Heliox10:

$$T_{1/2} = 15.5 \text{ min.}$$

stop times, NDL (= no decompression limits), time to flight etc.

We take equation (6a) and re-arrange a little bit:

$$e^{-k * t} = [(P_t(t) - P_{\text{alv}0}) / (P_{t0} - P_{\text{alv}0})] \quad (20)$$

The fraction may be the expression in the squared brackets [...]. We put the equation to logarithms:

$$\ln[...] = \ln(e^{-k * t}) = -k * t$$

We are solving for t:

$$t = -1 / k * \ln[...]$$

and from (12) with $k = \ln(2) / \tau$ we have the times t in question with:

$$t = -\tau / \ln 2 * \ln[(P_t(t) - P_{\text{alv}0}) / (P_{t0} - P_{\text{alv}0})] \quad (21)$$

With (21) and reasonable assumptions for the each of the pressure terms we are able to calculate now:

- the NDL with (17) and $P_{\text{amb}} = 1 \text{ Bar}$
- the time to flight, if we put in (17) the cabin pressure in the airplane to:
 $P_{\text{cabin}} = 0.58 \text{ Bar} = P_{\text{amb}}$
This is the somewhat lowest pressure, normally you have in civilian air planes during continental flights approximately 0.8 bar.
- the desaturation time, if we introduce a certain mathematical fuzziness:
 $P_t(t) = P_{N_2} + \Delta$, Delta being a very small number, because the straight line with constant $P = P_{N_2}$ and the exponential curve of the desaturation $P = P_t(t)$ will meet only at infinity, which, btw is a bit too long for a diving vacation...
Voellm suggests in [65], on p. 216, to take a certain part of the daily variation of air pressure, i.e. ca. + - 30 mbar.
- the required times on the deco stops
- (21) could be used as well for mixed gases only with reasonable assumptions for the respective a- and b- coefficients for each inert gas component in the breathing mix.

Buehlmann offers in [65], on p. 119:

$$P_t(t) = P_{t, \text{He}}(t) + P_{t, N_2}(t)$$

with the a- and b- coefficients being normalized with the respective partial pressures in each compartment (pls. cf. [54] on p. 86):

So for each compartment at any time t and for each combination of a- & b-values we have:

$$\begin{aligned} a(\text{He} + N_2) &= [(P_{t, \text{He}} * a_{\text{He}}) + (P_{t, N_2} * a_{N_2})] / (P_{t, \text{He}} + P_{t, N_2}) \\ b(\text{He} + N_2) &= [(P_{t, \text{He}} * b_{\text{He}}) + (P_{t, N_2} * b_{N_2})] / (P_{t, \text{He}} + P_{t, N_2}) \end{aligned} \quad (22)$$

(pls. cf. the examples in [4], p. 27 and as well in [5], p. 80.)

These are just basic remarks found in the sources and no one offers the full blown theory behind it.

[The ramifications and nuts & bolts you will find here!](#)

Gradient Factors (GF) / VGM (Variable Gradient Method)

The a- and b-coefficients are being embellished with the GF:

$$a \rightarrow a * GF \text{ and as well } b \rightarrow b / (GF - GF * b + b) \quad (22.a)$$

For all GF per compartment we have: $0 < GF \leq 1$
by calculating more conservatively.

If calculated more **aggressively** these GF become: **GF > 1**. Thus we end up at the VGM method, which allows this.

The conventional desktop deco softwares permit the input of 2 gradient factors: GF Hi and GF Lo.

GF Hi (= High) means the reduction of the leading M-value, or, as you would have it, the respective a- & b values pair, and thus resulting in a prolonged, last shallow stop.

GF Lo (= Low) results in a deeper, first stop.

With a simple linear relationship you can use the GFs automated over all required deco stages:

with: **GF Hi > GF Lo**

$$\begin{aligned} GF_m &= (GF_{Hi} - GF_{Lo}) / \text{first stop depth} \\ GF &= GF_{Hi} - GF_m * \text{actual stop depth} \end{aligned} \quad (22.b)$$

Within the **VGM (Variable Gradient Method)** framework you could, instead of using ONE PAIR of GF Hi/Lo for every compartment, use a different pair of GF Hi/Lo individually per EACH compartment.

VPM (Varying-Permeability Model)

The main source:

Yount DE, Hoffman DC. On the use of a bubble formation model to calculate diving tables. Aviat. Space Environ. Med. 1986; 57: 149 - 156.

More and other sources in the german version of: ["deco manual"](#)

$$\begin{aligned} r_1^{\min} &= 1 / [(1/r_0^{\min}) + (P_1 - P_0) / 2 * (\gamma_c - \gamma)] \\ r(t_r) &= r_1^{\min} + (r_0^{\min} - r_1^{\min}) * [1 - \exp(-t_r / \tau_r)] \\ P_{ss}^{\min} &= 2 * (\gamma / \gamma_c) * (\gamma_c - \gamma) / r(t_r) \\ P_{ss}^{\text{new}} &= [b + (b^2 - 4 * c)^{1/2}] / 2 \\ \text{mit:} \\ b &= P_{ss}^{\min} + \lambda * \gamma / [\gamma_c * (t_D + H / \ln(2))] \\ c &= (\gamma / \gamma_c)^2 * \lambda * (P_1 - P_0) / (t_D + H / \ln(2)) \end{aligned} \quad \begin{array}{l} \text{(Eq. 1 -> 4c)} \\ \text{on p. 150 -} \\ \text{151} \end{array}$$

These 5 "free" parameters have been established through numerical "best fits" of the TTS (time-to-surface) from the USN- and the RNPL-Tables:

$$\begin{aligned} \gamma_c &= 257 \text{ dyn / cm (= mN/m)} \\ \gamma &= 17,9 \text{ dyn / cm (= mN/m)} \\ r_0^{\min} &= 0,8 \mu\text{m} \\ \tau_r &= 20160 \text{ min (= 14 days)} \\ \lambda &= 7500 \text{ feet * min (ca. 227,3 Bar * min)} \end{aligned} \quad \text{(on p. 151 \& 155)}$$

RGBM folded over ZH-L

Wienke offers a simple method (in [31], p. 79 - 80; and in [71], p. 33 - 40) how to fold the results of the RGBM methods over an existing ZH-L model: it is simply a linear scaling of the a- and b- coefficients.

creative labels for it:

tiny RGBM

modified RGBM

recreational RGBM

Haldane-imbedded RGBM

This model you will find in all **RGBM-simulators** like:

GAP, ABYSS, etc.-softwares and as well in all MARES and SUUNTO computers.

This is accomplished with the so-called "**reduction factors**" **f**. In plain language:

the 16 ZH-L compartments and the corresponding HT with the central equation (17) is still used.

With these **f**, ($f < 1$), we will have (17):

$$\begin{aligned} P_{\min} &= (p - a_f) * b_f \\ \text{with:} \\ a_f &= a * f \\ \text{and:} \\ b_f &= b / f * (1 - b) + b \end{aligned}$$

f is dependant on the HT and defined only for air/nitrox for a HT > 180 min.

(REM:

On closer inspection this is just identical with (22.a))

As well all these factors which are missing up to now in the standard ZH-L model are ostensibly keyed in:

$$f = (1 - f_0) * \tau / 180 + f_0$$

with:

$$f < 1$$

and:

$$\tau > 180 \text{ min.}$$

For the factors: repetitive dives (rp), deeper than previous, i.e. reversed profiles (dp) and multiday-non-limit diving (dy) there is the following for f_0 :

$$f_0 = 0.45 * f_{rp} + 0.30 * f_{dp} + 0.25 * f_{dy}$$

This means:

rp = repetitive

dp = deeper than previous (reversed profile), i.e. shallower dive first

dy = multiday, diving frequency over a time span > 30 h

For these factors there are the following relationships:

$$f_{rp} = 1 - 0.45 \exp \left[- (t_{sur} - \eta_{rp})^2 / 4 * \eta_{rp}^2 \right] \quad (23)$$

$$f_{dp} = 1 - 0.45 * \left[1 - \exp \left(- \Delta P_{max} / P_{max} \right) \right] * \exp \left[- (t_{sur} - \eta_{dp})^2 / 4 * \eta_{dp}^2 \right]$$

$$f_{dy} = 0.70 + 0.30 \exp \left(- n / \eta_{dy} \right)$$

Variable	Definition
10 min. < η_{rp} < 90 min.	
30 min. < η_{dp} < 120 min.	
12 h. < η_{dy} < 18 h.	
t_{sur}	Surface Interval (SI) in [min.]
ΔP_{max}	the maximal pressure difference for reversed profiles
P_{max}	maximal ambient pressure
n	1. multi day-diving frequency within 24 h, p.36 2. number of diving-days within 30 h (???), p. 37

(REM:

1) the factor f_{dy} somehow fluctuates between 0.25 and 1.0 ... (p. 36)

2) n is obviously not defined soundly, as well the units (1/time or dimensionless)

do not fit to the units of η !!!)

3) as well over time there are some slightly incompatible changes in the definitions and parameters ...

--> in [31] from 1998, p.79

--> in [71] from 2003, p. 36 & 37 as well

--> in [144] from 2008, p. 351 & 372, and, finally:

--> in [175] from 2015, p. 159, 160 and p. 284 this looks **completely different !!!**)

$$\chi_1 = 1 - 0.45 \exp \left[- \frac{(t_{SI} - 30)^2}{900} \right]$$

$$\chi_2 = 1 - 0.45 \left[1 - \exp \left(- \frac{\delta}{P} \right) \right] \exp \left[- \frac{(t_{SI} - 60)^2}{3600} \right]$$

$$\chi_3 = 0.70 + 0.30 \exp \left(- \frac{n}{12} \right)$$

References: [31]; [71]; [144]; [175]:

[these: \[xyz\] refer to the entries in that list](#)

For helium we have analogous:

$$f = (1 - f_0) * \tau / 67.8 + f_0$$

with:

$$f < 1$$

and:

$$\tau > 67.8 \text{ min.}$$

For trimix we have for the corresponding partial pressures: $f_{O_2} + f_{N_2} + f_{He} = 1$.

For the complete inert gas pressure Π in the compartments we have:

$$\Pi = (P_{amb, N_2} + P_{amb, He}) + (P_{t, N_2} - P_{amb, N_2}) * e^{(-k_{N_2} * t)} + (P_{t, He} - P_{amb, He}) * e^{(-k_{He} * t)}$$

Now, however, the fractions f for the breathing gas are used for normalization: a_{f, N_2} and $a_{f, He}$ and as well b_{f, N_2} and $b_{f, He}$ in order to calculate the critical a, b - coefficients for the trimix:

$$A_f = (f_{N_2} * a_{f, N_2} + f_{He} * a_{f, He}) / (f_{N_2} + f_{He})$$

and, as well:

$$B_f = (f_{N_2} * b_{f, N_2} + f_{He} * b_{f, He}) / (f_{N_2} + f_{He})$$

and thus the critical pressure:

$$P_{min} = (\Pi - A_f) * B_f$$

(REM:

1) the formula for Π on p. 39 is wrong, but here we have it corrected!

2) Π is being calculated obviously in a different manner as per (22). The question is: what happens during a change of the deco mix with pure oxygen? the fractions of He and N_2 become identical to 0 in the breathing gas, and thus $A_f = 0$ and as well $B_f = 0$ and thus also $P_{min} = 0 \dots ???$)

For the inherent sub-saturation (aka: the oxygen window v) Wienke offers the following;

v [fsw], $P = P_{amb} = P_{absolut}$ [fsw], f_{O_2} oxygen fraction

$$v = f_{O_2} * P - 2.04 * (1 - f_{O_2}) - 5.47 \quad (24)$$

(REM:

But the oxygen window is non-linear, it reaches its maximum opening at ca. 1600 mm Hg! And stays there ...)

Relationship between k and the compartments

From (12) we have the following, purely mathematical: $\tau = \ln(2) / k$

resp.: $k = \ln(2) / \tau$

How is now the physiological relationship between k and the compartments, i.e. the various rates of perfusion and the various solubilities?

A very simple mass-balance for an inert gas within a compartment of tissue, solely feed by blood perfusion:

$$N_{2_{stored}} = N_{2_{in}} - N_{2_{out}}$$

The inert gas being transported via arterial blood to the tissue and leaving this region with the venous return.

Hereby it is assumed that the arterial (p_{N_a}) and the alveolar inert gas partial pressures (p_{N_A}) are equal, and that the

diffusion between adjacent capillaries is instantaneously, i.e.: as well the venous (p_{N_v}) and the compartment partial pressure (p_{N_t}) are equal for this particular inert gas.

The solubility coefficients for this inert gas for blood (b) and tissue (t) are: α_b and as well α_t .

Q is the blood flow, V_t is the tissue volume. Starting at time $t = 0$ it is assumed that p_{N_a} reaches immediately a constant value, p_a .

The rate of change of the pressure in the compartment dictates the rate of change for storage in this compartment.

The little drawing may help. We have the following:

$$\alpha_t * V_t * dp_t/dt = \alpha_b * dQ/dt * p_{N_a} - \alpha_b * dQ/dt * p_{N_v} \quad (25)$$

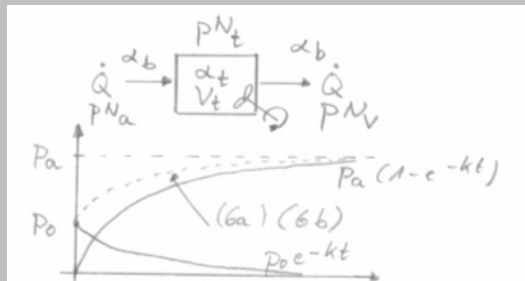
with: $dp_t/dt + k * p_t = k * p_a$

as well: $k = \alpha_b * dQ/dt / (\alpha_t * V_t)$

The solution of (25) is either (6a) or (6b). Thus we have in the end the relationship between the HT and the tissue perfusion and the solubilities:

$$\tau = 0.693 / k = 0.693 / (\alpha_b * dQ/dt / \alpha_t * V_t) \quad (26)$$

The total partial pressure over time ($p_t(t)$) is the sum of of the decrease of p_0 and the increase of the alveolar pressure p_{N_A} :



The small stirrer is symbolizing that we have a "well stirred" compartment

VVAL18 LEM Model

Through the revision 6 from 04/2008 the USN table has been modified from a Haldane/Workman EE model to a LE- resp. LEM model.

EE means: exponential for saturation AND desaturation

LE means: exponential for saturation, linear for desaturation

LEM means: exponential for saturation, linear for desaturation, M for multigas

At first the arterial inert gas partialpressure p_a is corrected:

$$p_{a,N_2} = P_{amb} - P_{I,O_2} - 1.5 \quad (30)$$

p_{a,N_2} = arterial inert gas partialpressure

P_{amb} = absolut, i.e. ambient pressure

P_{I,O_2} = inspiratory oxygen partialpressure (in FSW)

1.5 = arterial CO_2 partial pressure in FSW (35 mmHg)

The partial pressure of water vapor is ignored for the time being .

Starting from equation (6a) a linear behavior is defined for the desaturation.

This linear desaturation is being calculated until the "**exponential cross over point**" is reached, after that we have the usual exponential behavior.

Is for any time in any compartment during the desaturation-phase the inert gas-partial pressure greater than the ambient pressure the following linear equation has to be used:

$$P_{T,N_2} = P_{Ti,N_2} + (2.8 - P_{I,O_2}) * K * T \quad (31)$$

P_{T,N_2} = inert gas partialpressure in compartment at time T

P_{Ti,N_2} = initial inert gas partialpressure in compartment

T = time on actual depth stage

2.8 = sum of venous O_2 (46 mmHg) and CO_2 (53 mmHg) minus arterial CO_2 (35 mmHg) = 64 mmHg in FSW

equation (31) has to be used if:

$$P_{T,N_2} \geq P_{amb} - 4.3 \quad (32)$$

with 4.3 = sum of venous O_2 and CO_2 in FSW (=99 mmHg).

The criteria for safe ascent are no longer called M-Values but instead:

MPTT = Maximum Permissible Tissue Tensions.

[You will find various MPTT-tables there....](#)

The calibration for mixed gases and as well the adaption via the NMRI model is taking place according to further parameters of gases in the blood.

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The coefficients of various perfusion models

M-Values USN / Workman (1965) for N_2

Halftime [min]	M_0 - Value [fsw = feet of sea water]	Delta M ΔM , [fsw/ft]
5	104	1,8
10	88	1,6
20	72	1,5
40	56	1,4
80	54	1,3
120	52	1,2
160	51	1,15
200	51	1,1
240	50	1,1

M-Values USN / Workman (1965) for He

Halftime [min]	M_0 - Value [fsw = feet of sea water]	Delta M ΔM , [fsw/ft]
5	86	1,5
10	74	1,4

20	66	1,3
40	60	1,2
80	56	1,2
120	54	1,2
160	54	1,1
200	53	1,0
240	53	1,0

MPTT-Values USN VVAL18 / Ed Thalmann (2008) for N₂

Halftime [min]	MPTT - Value [fsw = feet of sea water] für Tiefe = 10 feet	Delta M ΔM , [fsw/ft]
5	120	1,0
10	98	1,0
20	78	1,0
40	56	1,0
80	48,5	1,0
120	45,5	1,0
160	44,5	1,0
200	44	1,0
240	43,5	1,0

MPTT-Values USN VVAL76 - 1 / Ed Thalmann for N₂

Halftime [min]	MPTT - Value [fsw = feet of sea water] für Tiefe = 10 feet	Delta M ΔM , [fsw/ft]
5	106,7	1,0
10	86,7	1,0
20	70,3	1,0
40	56	1,0
80	48,5	1,0
120	45,5	1,0
160	44,5	1,0
200	44	1,0
240	43,5	1,0
255	40,4	1,0

Decompression Computation and Analysis Program, Dave Kenyon / R. W. Hamilton

DCAP MF11F6 M-Values for N₂ (1975)

Compartment #	1	2	3	4	5	6	7	8	9	10	11
HT [min]	5	10	25	55	95	145	200	285	385	520	670
M ₀ [fsw]	104,0	80,5	62,3	48,6	45,4	44,7	44,1	44,0	44,0	44,0	43,5
ΔM	1,30	1,05	1,08	1,06	1,06	1,02	1,01	1,0	1,0	1,0	1,0

ORCA EDGE M₀ (1981)

Compartment #	1	2	3	4	5	6	7	8	9	10	11	12
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HT [min]	5	11	17	24	37	61	87	125	197	271	392	480
M ₀ [fsw]	100,0	81,8	71,5	63,7	55,9	50,7	46,8	43,0	39,1	36,5	33,9	33,0

PADUA (Pennsylvania Analysis of Decompression for Undersea and Aerospace) (1976)

Compartment #	1	2	3	4	5	6	7	8	9	10
HT [min]	5	10	20	40	80	120	160	240	320	480
M ₀ [fsw]	100,00	84,00	68,00	53,00	52,00	51,00	50,00	49,00	49,00	48,00
Δ M	1,60	1,50	1,40	1,30	1,20	1,10	1,10	1,0	1,0	1,0

Hans Hass Deco-Brain II (Version P2-2, 1985)

Compartment #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
HT [min]	4	7,94	12,2	18,5	26,5	37	53	79	114	146	185	238	304	397	503	635
a [Bar]	1,8917	1,4699	1,0575	0,87828	0,69207	0,53783	0,43429	0,40135	0,41385	0,43085	0,43170	0,37321	0,25083	0,25086	0,25087	0,25088
b [-]	0,82593	0,82604	0,83132	0,84232	0,85687	0,88250	0,89375	0,91737	0,90848	0,94176	0,94122	0,94896	0,96671	0,96668	0,96666	0,96666

M₀-Values RDP (for N₂, 1987)

Comp. #	1	2	3	4	5	6	7	8	9	10	11	12	13	14
HT [min.]	5	10	20	30	40	60	80	100	120	160	200	240	360	480
M ₀ [fswa]	99,08	82,63	66,89	59,74	55,73	51,44	49,21	47,85	46,93	45,78	45,07	44,60	43,81	43,40

ZH-L Values

Nitrogen (N₂) HT, a- & b Coefficients (1983)

Compartment #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
HT [min]	2,65	7,94	12,2	18,5	26,5	37	53	79	114	146	185	238	304	397	503	635
a	2,2	1,5	1,05	0,9	0,75	0,60	0,45	0,43	0,43	0,43	0,43	0,35	0,23	0,23	0,23	0,23
b	0,820	0,820	0,825	0,835	0,845	0,860	0,870	0,890	0,890	0,931	0,931	0,943	0,962	0,962	0,962	0,962

(aus [4], S. 27)

Helium (He) HT, a- & b Coefficients (1983)

Compartment #	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
HT [min]	1	3	4,6	7	10	14	20	30	43	55	70	90	115	150	190	240
a	2,2	1,5	1,05	0,9	0,75	0,60	0,45	0,43	0,43	0,47	0,47	0,47	0,47	0,47	0,47	0,47
b	0,82	0,82	0,825	0,835	0,845	0,860	0,870	0,890	0,890	0,925	0,925	0,925	0,925	0,925	0,925	0,925

(from [4], p. 27, resp. [65], p. 158)

Helium (He) HT, a- & b Coefficients (1993)

Compartment #	1	1b	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
HT [min]	1,51	1,88	3,02	4,72	6,99	10,21	14,48	20,53	29,11	41,20	55,19	70,69	90,34	115,29	147,42	188,24	240,03
a	1,7424	1,6189	1,3830	1,1919	1,0458	0,9220	0,8205	0,7305	0,6502	0,5950	0,5545	0,5333	0,5189	0,5181	0,5176	0,5172	0,5119

b	0,4254	0,4770	0,5747	0,6527	0,7223	0,7582	0,7957	0,8279	0,8553	0,8757	0,8903	0,8997	0,9073	0,9122	0,9171	0,9217	0,9267
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(from [5], p. 109, resp. [65], p. 158)

Nitrogen (N₂) HT, a- & b Coefficients (1993)

Compart. #	HT	ZHL-16 A	ZHL-16 A	ZHL-16 B	ZHL-16 C
No.	[min.]	b theoretical	a theoretical	a	a
1	4,0	0,5050	1,2599	1,2599	1,2599
1b	5,0	0,5578	1,1696	1,1696	1,1696
2	8,0	0,6514	1,0000	1,0000	1,0000
3	12,5	0,7222	0,8618	0,8618	0,8618
4	18,5	0,7825	0,7562	0,7562	0,7562
5	27,0	0,8126	0,6667	0,6667	0,6667
6	38,3	0,8434	0,5933	0,5600	0,5403
7	54,3	0,8693	0,5282	0,4947	0,4410
8	77,0	0,8910	0,4701	0,4500	0,4000
9	109,0	0,9092	0,4187	0,4187	0,3750
10	146,0	0,9222	0,3798	0,3798	0,3500
11	187,0	0,9319	0,3497	0,3497	0,3295
12	239,0	0,9403	0,3223	0,3223	0,3065
13	305,0	0,9477	0,2971	0,2850	0,2835
14	390,0	0,9544	0,2737	0,2737	0,2610
15	498,0	0,9602	0,2523	0,2523	0,2480
16	635,0	0,9653	0,2327	0,2327	0,2327

(from [5], p. 108, resp. [65], p. 158)

ZH-L Parameters / Wienke (2003) for N₂

Halftime [min]	a [Bar]	b
5,0	1,198	0,542
10,0	0,939	0,687
20,0	0,731	0,793
40,0	0,496	0,868
65,0	0,425	0,882
90,0	0,395	0,900
120,0	0,372	0,912
150,0	0,350	0,922
180,0	0,334	0,929
220,0	0,318	0,939
280,0	0,295	0,944
350,0	0,272	0,953
450,0	0,255	0,958
635,0	0,236	0,966

ZH-L Parameters / Wienke (2003) for He

Halftime [min]	a [Bar]	b
1,8	1,653	0,461
3,8	1,295	0,604

7,6	1,008	0,729
15,0	0,759	0,816
24,5	0,672	0,837
33,9	0,636	0,864
45,2	0,598	0,876
56,6	0,562	0,885
67,8	0,541	0,892
83,0	0,526	0,901
105,5	0,519	0,906
132,0	0,516	0,914
169,7	0,510	0,919
239,6	0,495	0,927

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Mixed Gas Theory

If we have more than only one inert gas (nitrogen N₂ with air/nitrox), for e.g. two as in the case of trimix / heliox (nitrogen N₂ and helium He) we have to expand the Haldane-equation (6a).

Basically the law of Henry will be valid for each inert gas_i separately:

(well, well, except the region, what specialists call "Poyinting effect", i.e.: very high pressures and very complex organic molecules ...)

$$q_i = l_i * p_i * V_k$$

$$Q = \sum q_i \quad (50)$$

The sum encloses all inert gases_i and as well all compartments_k

Variable	Definition
q _i	amount of diluted inert gas i within a compartment with volume V _k
l _i	solubility coefficient of inert gas _i valid for this volume V _k
p _i	partial pressure of inert gas _i
V _k	volume of compartment k

For the sum of all partial pressures we will have the law of Dalton:

$$P = \sum p_i = p_1 + p_2 + p_3 + \dots \quad (51)$$

Thus we will have for a stationary state in a compartment the following:

$$P_t(0) = \sum P_{t,i}(0) = P_{t,N_2}(0) + P_{t,He}(0) \quad (52a)$$

as well for the alveoli:

$$P_{alv}(0) = \sum P_{alv,i}(0) = P_{alv,N_2}(0) + P_{alv,He}(0) \quad (52b)$$

as well for all times t:

$$P_{tiss}(t) = P_t(t) = \sum P_{tiss,i}(t) = P_{tiss,N_2}(t) + P_{tiss,He}(t) \quad (52c)$$

Thus we expand the Haldane-equation (6a):

$$P_t(t) = P_{alv0,N_2} + [P_{t0,N_2} - P_{alv0,N_2}] e^{-k_{N_2}t} + P_{alv0,He} + [P_{t0,He} - P_{alv0,He}] e^{-k_{He}t} \quad (53)$$

We have to check if this expansion will satisfy the basic differential equation (1a). So we expand (1a) with an additional term for a second inert gas:

$$dP_t(t)/dt = k_1 * [P_{alv,N_2}(t) - P_{t,N_2}(t)] + k_2 * [P_{alv,He}(t) - P_{t,He}(t)] \quad (54)$$

We determine the proportionality factors k₁ and k₂, by calculating d P_t(t) / dt from (53) :

$$dP_t(t)/dt = -k_{N_2} * [P_{t0,N_2} - P_{alv0,N_2}] e^{-k_{N_2}t} - k_{He} * [P_{t0,He} - P_{alv0,He}] e^{-k_{He}t} \quad (53a)$$

... and with this terms we replace the left hand side of (54):

$$\begin{aligned}
 & -k_{N_2} * [P_{t0,N_2} - P_{alv0,N_2}] e^{-k_{N_2}t} - k_{He} * [P_{t0,He} - P_{alv0,He}] e^{-k_{He}t} = \\
 & k_1 * [P_{alv,N_2}(t) - P_{t,N_2}(t)] + k_2 * [P_{alv,He}(t) - P_{t,He}(t)] = \quad (55) \\
 & k_1 * P_{alv,N_2}(t) - k_1 * P_{t,N_2}(t) + k_2 * P_{alv,He}(t) - k_2 * P_{t,He}(t)
 \end{aligned}$$

For the $P_{t,i}(t)$ we exploit once again (52c) resp. (53), put this into (55), right hand side, re-arrange and receive:

$$\begin{aligned}
 & -k_{N_2} * [P_{t0,N_2} - P_{alv0,N_2}] e^{-k_{N_2}t} - k_{He} * [P_{t0,He} - P_{alv0,He}] e^{-k_{He}t} = \\
 & k_1 * P_{alv,N_2}(t) + k_2 * P_{alv,He}(t) - k_1 * [P_{alv0,N_2} + [P_{t0,N_2} - P_{alv0,N_2}] e^{-k_{N_2}t} - k_2 * [P_{alv0,He} + [P_{t0,He} - P_{alv0,He}] e^{-k_{He}t} \quad (55a)
 \end{aligned}$$

We multiply the terms of the squared brackets [...] of the right hand side and re-arrange according to constant resp. time-dependant terms:

$$\begin{aligned}
 & -k_{N_2} * [P_{t0,N_2} - P_{alv0,N_2}] e^{-k_{N_2}t} - k_{He} * [P_{t0,He} - P_{alv0,He}] e^{-k_{He}t} = \\
 & k_1 * P_{alv,N_2}(t) + k_2 * P_{alv,He}(t) - k_1 * P_{alv0,N_2} - k_1 * (P_{t0,N_2} - P_{alv0,N_2})e^{-k_{N_2}t} - k_2 * P_{alv0,He} - k_2 * (P_{t0,He} - P_{alv0,He})e^{-k_{He}t} = \quad (55b) \\
 & k_1 * [P_{alv,N_2}(t) - P_{alv0,N_2}] + k_2 * [P_{alv,He}(t) - P_{alv0,He}] - k_1 * (P_{t0,N_2} - P_{alv0,N_2})e^{-k_{N_2}t} - k_2 * (P_{t0,He} - P_{alv0,He})e^{-k_{He}t}
 \end{aligned}$$

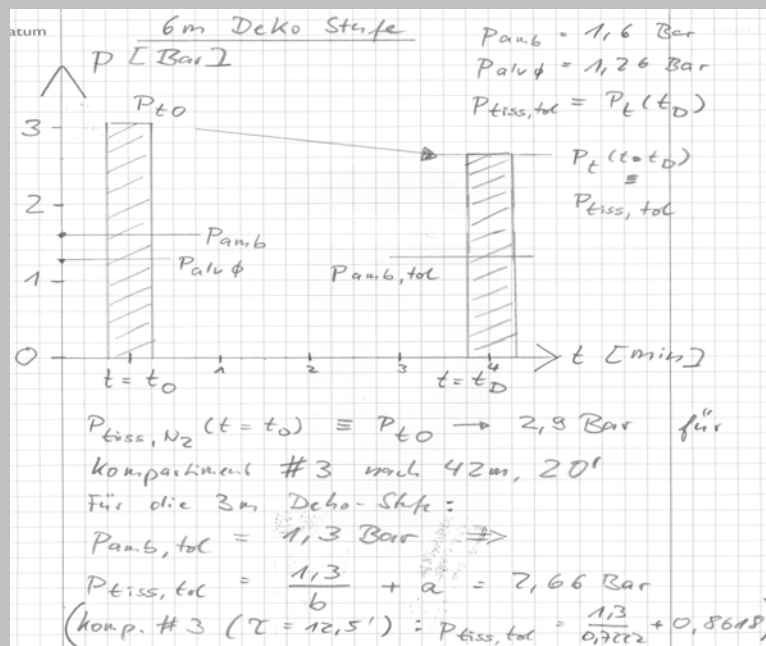
We take now the identical boundary conditions for the both inert gases which guided us to the solution of (6a):

$$\begin{aligned}
 & P_{alv,N_2}(t) = P_{alv0,N_2} \text{ for all times } t \text{ and analogous} \\
 & P_{alv,He}(t) = P_{alv0,He} \\
 & \text{the equation (55a) will be identically satisfied with:} \quad (55c) \\
 & k_1 = k_{N_2} \text{ as well: } k_2 = k_{He}
 \end{aligned}$$

Thus the approach (53) satisfies the central differential equation (1a), if the second gas is applied as purely additive: this yields for the stationary part, i.e. the part with no variation in time in the alveolar gas (52b) and as well for the time-dependant part (52c).

Calculation of decompression times with mixed gases

To start with, we check the very simple situation with only one inert gas (air/nitrox) on a definite deco stage (6m), before we could ascent to the next one (3m):



NB: the figures are only a first, coarse approximation: the identification of the variables is for getting an overview over equation (21):

$$t = -\tau / \ln 2 * \ln [(P_t(t) - P_{alv0}) / (P_{t0} - P_{alv0})]$$

The actual deco time t_d is the maximal value out of all calculated deco times which have been assessed for the complete group of all the compartments.

For the terms we have the following:

- $P_t(t)$: pressure in compartment/tissue at the end of the deco-time t_d , it is calculated via the
- P_{amb} , tolerated for the desired deco-stage.
- With $P_{amb,tol}$ the tolerated tissue-pressure $P_{tiss,tol}$ is calculated with: the equation (17):

$$P_{t,tol,ig} = P_{amb} / b + a$$

These a- & b-values are constants and could be looked up in a table. Thus the $P_t(t)$ can be calculated for every compartment.

- P_{alv0} : this is the alveolar pressure of the respective inert gas, at start of deco, basically determined by the ambient pressure, embellished with the fraction f of the inert gas; (let's forget R_q etc. ... for the time being):
- $P_{alv0} = P_{alv0}(t_0) = P_{ambient} * f_i$
- P_{t0} : compartment pressure at start of deco, at $t = t_0$
i.e.: the sum over the complete diving-profile up to t_0 .

The extensions for **mixed gases** with (22) are for the a- und b-coefficients, and as well the sets for τ resp. λ (equation set 56)!

$$\begin{aligned} a^* &= a(\text{He} + \text{N}_2) = [(P_t, \text{He} * a_{\text{He}}) + (P_t, \text{N}_2 * a_{\text{N}_2})] / (P_t, \text{He} + P_t, \text{N}_2) \\ b^* &= b(\text{He} + \text{N}_2) = [(P_t, \text{He} * b_{\text{He}}) + (P_t, \text{N}_2 * b_{\text{N}_2})] / (P_t, \text{He} + P_t, \text{N}_2) \\ \lambda^* &= \lambda(\text{He} + \text{N}_2) = [(P_t, \text{He} * \lambda_{\text{He}}) + (P_t, \text{N}_2 * \lambda_{\text{N}_2})] / (P_t, \text{He} + P_t, \text{N}_2) \end{aligned} \quad (56)$$

and this is for all the k compartments, e.g.: $k = 1 - 16!$ (within the standard ZH-L framework; if you want to compare other things, like the ANALYST software from COCHRAN, k runs from 1 to 20, that this from approx. 1.25 min to 900 mins.)

As well we have for our two inert gases He and N_2 (52a, 52b, 52c) and thus:

- $P_{alv0} = P_{alv0, \text{N}_2}(t_0) + P_{alv0, \text{He}}(t_0)$
- $P_{t0} = P_{\text{N}_2, t0} + P_{\text{He}, t0}$
- for the tolerated tissue pressure $p_{\text{tissue, tolerated}}$ we have now analogous:

$$P_{t, \text{tol. ig}} = P_{\text{amb}} / b^* + a^*$$

If you take now for the calculation of the deco times t_d the equation (21) with all of the above cited extensions for mixed gases, you generate an equation with two more unknowns,

i.e.: $p_{\text{tiss, N}_2}(t)$ und $p_{\text{tiss, He}}(t)$ for the time $t = t_d$

because of: $P_t(t) = p_{\text{tiss, N}_2}(t) + p_{\text{tiss, He}}(t)$, und this for all compartments! As

a^* , b^* and as well λ^* have been assessed for time $t = t_0$, but this is only the start of the deco phase!

At first, t_d is unknown and thus as well the respective compartment saturations $p_{\text{tiss}}(t_d)$ and thus consequently also a^* , b^* , λ^* at time $t = t_d$.

- 1) numerical method:

In clear language: in dealing with two (2, or more) inert gases, we have to solve (21) either approximatively by taking small incremental time-steps like Δt , e.g.: $\Delta t = 1$ sec, and checking the new

$P_t(t + \Delta t) = p_{\text{tiss, N}_2}(t + \Delta t) + p_{\text{tiss, He}}(t + \Delta t)$, derived from the previous $P_t(t)$, and comparing the

tolerated ambient pressure $P_{\text{ambient, tolerated}}$ of the desired depth / deco-stage depth and the $P_{\text{tissue, tolerated}}$ in question: is after n time-steps

$P_t(t = t_d) = P_t(t_0 + n * \Delta t) \leq P_{\text{tiss, tol}}$, we have found with $t = t_d$ the required deco-time.

Further Possibilities:

- 2) "quasi"-analytical method:

we accept **an error**, by taking the well-known t_0 from the start of the deco:

$$P_t(t_0) = p_{\text{tiss, N}_2}(t_0) + p_{\text{tiss, He}}(t_0)$$

and the respective weighting of the a-, b-coefficients and λ (equations 56) and simply using all of these for the calculations of $P_t(t)$ for all times t .

This error produces at first an over-estimation of the deco-times!

(in dependancy of: depth, time and He-fraction)

(But it seems, that this approach is used in a couple of desktop deco-sofwares and regular mixed gas diving computers: the numerical calculations (method 1) require more efforts during programming and as well additional variables, which means more memory. As well it is time-consuming, dependant on the amount of time-steps and the step-size of time-steps: each compartment has to go through long loops for each deco-stage.)

- 3)

We try a Taylor-Expansion in Series for equation (21):

we take equation (53) and re-arrange analogous with (21):

$$\begin{aligned} P_t(t) &= P_{alv0, \text{N}_2} + P_{alv0, \text{He}} + [P_{t0, \text{N}_2} - P_{alv0, \text{N}_2}] e^{-k_{\text{N}_2} t} + [P_{t0, \text{He}} - P_{alv0, \text{He}}] e^{-k_{\text{He}} t} \\ &= P_{\text{amb}} / b^* + a^* \end{aligned}$$

Now we are using the Taylor expansion for the exponential term within the $P_t(t)$

with $x = -k_i * t$, and for e^x :

$$e^x \cong 1 + x + (x^2 / 2) + (x^3 / 6) + \dots + \dots$$

We stop the Taylor expansion after the 3rd or even already after the 2nd term: this is sufficiently accurate for small x , that is: short deco stops!

The $P_t(t)$ in dependancy with the b^* and a^* means that the time t appears as well in the numerator AND the denominator and thus we get a cubic or bi-quadratic polynomial for t in its fullest beauty. This one we have to solve as well numerically. But thus we wouldn't gain anything in comparison to method 1!

- 4)

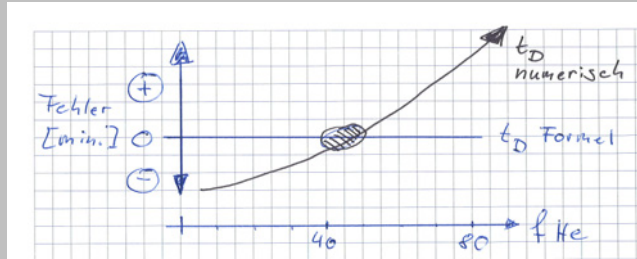
Now we would take a couple of examples (*) and check the error produced with the prevailing method 2, by comparing with the "real" deco-times calculated with method 1; we did so for various square depth-profiles and various mixes for the bottom-gas. We compare thus the deco times from

two commercially available desktop deco softwares and DIVE V3 with the times assessed by method 1.

(*) a couple here means exactly: more than 1,900 profiles from the TEC-recreational set.

We took 6 depth profiles from 30 - 80 m, 5 bottom times from 20 - 60 min., and helium- fractions from 5 - 80 % (16 mixes), over all 480 profiles, and the whole then 4 (four!) times ...

This error increases with the decompression obligation, i.e. with increasing diving-depth and bottom-time and the dependency on the helium fractions f_{He} is, roughly speaking like this, only qualitative, sketch:



We are seeing the deco-times t_D sketched over the helium-fractions f_{He} :

the deviations are the differences from equation (17) (= method 2) and the mean-values out of all the 3 softwares in comparison to the numerical values according method 1.

I.e. as pointed out above: in the beginning we have this overestimation of deco times (error < 0), in the region of the regular TEC-REC trimixes, especially at around 35 - 45 % He the error converges $\rightarrow 0$, and, by increasing f_{He} is growing steadily (error > 0).

special sources:

- CAISSON 03, 2011: Dekompressionsberechnungen für Trimix-Tauchgänge mit PC-Software: Reparieren Gradientenfaktoren defekte Algorithmen oder defekte Software-Implementierungen? [CAISSON 26. Jg./2011/Nr. 3, S. 4 - 12](#)
- Tech Diving Mag, Issue 05 / 2011: Decompression calculations for trimix dives with PC software; gradient factors: do they repair defective algorithms or do they repair defective implementations? [TDM, Ausgabe 05 / 2011, S. 41 - 53](#)
- International Journal of the Society for Underwater Technology, November 2012: Variations in the TTS: where do they come from? [SUT, Vol. 31, No. 1, pp. 43 - 47, 2012](#)
- more sources from the [Tauchsportcenter Esslingen](#):
- the [deco workshop](#)
- as well the [accompanying manual](#)

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